Quiz 14

March 24, 2017

Show all work and circle your final answer.

Determine whether the following series are absolutely convergent, conditionally convergent, or divergent. State the test you are using and check all necessary conditions.

1. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

conditional convergence: use AST. $\frac{1}{\ln n}$ decreases to 0, so $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ converges.

absolute convergence: use DCT with $\sum_{n=2}^{\infty} \frac{1}{n}$, which diverges by p-series, $p=1$

$\frac{1}{\ln n} > \frac{1}{n}$ and $\frac{1}{\ln n} > 0$, $\frac{1}{n} > 0$ for $n \geq 2$, so diverges

$\boxed{\text{conditionally convergent}}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-9)^n}{n10^{n+1}}$

abs. convergence: use ratio test

$$\lim_{n\to\infty} \left| \frac{(-9)^{n+1}}{(n+1)10^{n+2}} \cdot \frac{n10^{n+1}}{(-9)^n} \right| = \lim_{n\to\infty} \left| \frac{9n}{10(n+1)} \right| = \frac{9}{10} < 1$$

so the series is $\boxed{\text{absolutely convergent}}$

(can also use DCT with $\sum_{n=1}^{\infty} \frac{9^n}{10^{n+1}}$, which converges by geom. series)

3. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{10^n}$

Notice all of the terms are positive, so the series is absolutely convergent or divergent.

use ratio test: $\displaystyle\lim_{n\to\infty} \left| \frac{(n+1)!}{10^{n+1}} \cdot \frac{10^n}{n!} \right| = \lim_{n\to\infty} \left| \frac{n+1}{10} \right| = \infty > 1,$

so the series $\boxed{\text{diverges}}$.

(can also use test for divergence)